Fast Reconstruction of Exact Maxwell Dynamics from Sparse Data

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The problem

From a few measurements of the electromagnetic fields **E** and **B** in space-time, reconstruct them **everywhere**, while obeying Maxwell's equations **exactly**.

$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} \qquad \nabla \cdot \mathbf{E} = 0$$
$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

Why it matters

RF & antennas
Radiation patterns from a few sensors

Tomography & imaging
Fast reconstruction from limited probes

Plasma diagnostics
Real-time fields in fusion reactors

SciML's "holy grail"
ML speed with physics-grade reliability

How does it compare?

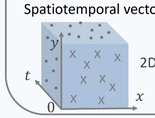
	Zero PDE residual	Works from sparse data	Under a minute to 5%	5% on all ground truths
FEM / FDTD numerical solvers	✗	✗	✗	✗
PINNs physics in the loss	✗	✓	✗	✗
S-EPGP exact-prior GPs	✓	✓	✗	✗
FLASH-MAX (ours) exact neural network	✓	✓	✓	✓

Neural operators *(FNO, DeepONet) amortize over many problem instances and need large simulated training sets: a different regime, so not compared here.*

FLASH-MAX

Fast-**L**earning **A**ccurate **S**olver for **H**omogeneous **MAX**well equations

Setting: Sparse Supervision
Inputs:
+ **Sparse** observations at $t = 0$;
+ **Optional** boundary observations
Output:
Spatiotemporal vector fields $\boldsymbol{E}, \boldsymbol{B}$

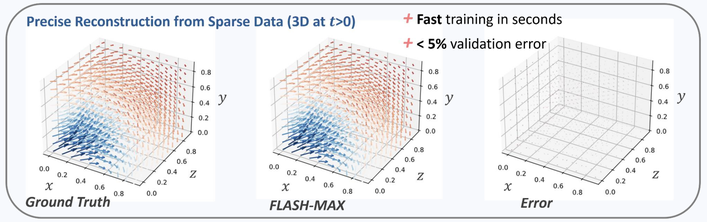


***FLASH-MAX:* Maxwell Solutions as the Hypothesis Class**
Each hidden neuron is an exact Maxwell solution
Learnable weights and biases: $\boldsymbol{w}, \boldsymbol{b}$
Customized Hidden Layer $\boldsymbol{z} = (\pm\sqrt{z_1^2 + z_2^2 + z_3^2}, z_1, z_2, z_3)$

x → *Hidden Layer* $\sigma(\boldsymbol{x}^T\boldsymbol{z} + \boldsymbol{b})\boldsymbol{p}(\boldsymbol{z})$ → *Outer Linear Layer* $\boldsymbol{w}^T\sigma(\boldsymbol{x}^T\boldsymbol{z} + \boldsymbol{b})\boldsymbol{p}(\boldsymbol{z})$ → $(E(x), B(x))$

+ ***Residual Error = 0*** *by construction*
+ ***Universal:*** *Expressive despite exactness*

Precise Reconstruction from Sparse Data (3D at $t>0$)
+ **Fast** training in seconds
+ **< 5%** validation error



Every hidden neuron is an exact Maxwell solution, so the predicted field satisfies the equations throughout training, and sparse data only has to select which solutions to superpose.

The architecture

$$\left(\widehat{\mathbf{E}}(\mathbf{x}), \widehat{\mathbf{B}}(\mathbf{x})\right) = \sum_{k=1}^{W}\sum_{i=1}^{2} \underbrace{w_{ki}}_{\text{amplitude}} \underbrace{\mathbf{p}_i(\mathbf{z}_{ki})}_{\text{polarization}} \underbrace{\sigma(\mathbf{x}\cdot\mathbf{z}_{ki} + b_i)}_{\text{wave profile}}$$

where $\mathbf{x} = (t, x, y, z)$, $\mathbf{z} = (z_0, z_1, z_2, z_3)$ with

$$\begin{cases} z_0^2 = z_1^2 + z_2^2 + z_3^2, \\ \mathbf{p}_1(\mathbf{z}) = (-z_1 z_3,\ -z_2 z_3,\ z_0^2 - z_3^2,\ -z_0 z_2,\ z_0 z_1,\ 0), \\ \mathbf{p}_2(\mathbf{z}) = (z_1 z_2,\ -z_0^2 + z_2^2,\ z_2 z_3,\ -z_0 z_3,\ 0,\ z_0 z_1). \end{cases}$$

Intuition Each neuron is an electromagnetic wave; training chooses which waves to superpose.
Training Plain MSE on the data: no PDE loss needed, the residual is 0 by construction.

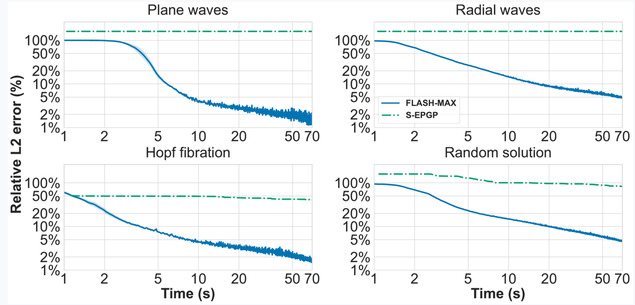
1 Race to 5% error

Ground truth	Boundary data	FEM	PINN	S-EPGP	FLASH-MAX
Plane waves	with	8.2%	304.2	27%	8.1
	without	–	119.3	25%	6.8
Radial waves	with	65%	94%	n/c	58.9
	without	–	87%	6%	11.9
Hopf fibration	with	157	235.3	37%	7.4
	without	–	89.8	153.3	3.2
Random solution	with	13,701	5.7%	75%	58.3
	without	–	131.9	516.7	6.1

Seconds to reach 5% validation error · **red** = never reached (best error shown); n/c = did not converge; – = FEM needs boundary data

FLASH-MAX: 3–59 s: 17–235× faster than any baseline that reaches 5%, and the only method to get there on Radial waves.

2 Accuracy in 70 seconds



FLASH-MAX reaches ≈1–5% error on every ground truth within 70 s; S-EPGP lags far behind. Only runs with residual < 0.01 are shown; FEM and PINN never qualify.

3 How little data is needed?

Minimum relative L² error (%) of FLASH-MAX

# points	Plane	Radial	Hopf	Random
100	2.6	7.4	<1.0	2.0
200	<1.0	2.9	1.1	<1.0
500	<1.0	<1.0	<1.0	<1.0
1,000	<1.0	<1.0	<1.0	<1.0
5,000	<1.0	<1.0	1.5	<1.0
12,000	1.3	<1.0	<1.0	<1.0

100 points
single-digit error on every ground truth (worst 7.4%)

500 points
all four ground truths below 1% error

Zero residual
Maxwell holds exactly, by construction

3–59 s
to 5% error: up to 235× faster

<1% error
from 500 points; single-digit from 100

Universal
can approximate any Maxwell solution

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